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CAPTURE OF CONVECTIVE VORTICES BY AN UNSTEADY-STATE FLOW IN A HELE–SHAW CELL

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The specific transient regime of thermal convection in a Hele–Shaw cell has been studied experimentally and theoretically. It is a kind of unsteady-state, four-vortex flow, symmetric about the vertical axis, in the form of periodic pulsations of the lower convective cells, with the latter being partially adsorbed by the upper vortices. The transient pulsational regimes in a Hele–Shaw cell were revealed experimentally in studying convective motions in a transformer oil. These pulsational flows are unstable and with time they are always rearranged into a steady four-vortex vibrational regime with reconnection of corner vortices.

Introduction. In cavities that offer a high hydrodynamic resistance, convective regimes need a rather long time to be established. Under certain conditions, the process of flow development is observed on successive replacement of some transient regimes by others. Unlike steady-state flows, transient convective regimes are unstable; they have their intensity, structure, and specific features of heat transfer, which should be taken into account in constructing various technical facilities in which allowance for thermal convection is required. This leads to the necessity of a thorough theoretical and experimental study of transient flows.

The cavity considered has the shape of a rectangular parallelepiped, one of the horizontal dimensions of which is much smaller than the other two. As is shown in experiments and theoretical calculations [1, 2], in this case the planes of the trajectories of the fluid elements are located parallel to the wide faces of the layer in a large range of inclination angles (0^o–75^o). In the literature, the thin vertical layer in which such kind of flows are observed is called a Hele–Shaw cell. Because of the high thermal and hydraulic resistances, the region of vibrational regimes in such a cell appears to be much closer to the lower level of instability than in a horizontal layer or cavities having dimensions of the same order of magnitude. Such a cavity is convenient for investigating by both experimental and theoretical methods by using an approximation of plane trajectories. We note that previously transient convective regimes in the Hele–Shaw cell have practically not been studied.

In [3], in numerical investigation of thermal convection in a Hele–Shaw cell, heated from below, with perfectly conducting wide faces, stable stationary and vibrational regimes were studied, the boundaries of the equilibrium stability for different side ratios of the wide faces of the cavity were determined, the structure of steady-state and vibrational flows was investigated, and it was also shown that with increase in the supercriticality regular vibrations become stochastic. We note that the indicated irregular vibrations were among the first physical realizations of chaotic behavior in a simple hydrodynamic system. Later, in [4] results of a more thorough investigation of thermal convection in a Hele–Shaw cell with wide boundaries made from plexiglas were presented. Apart from the fact that the experimental and theoretical results of [3] have been confirmed, new stable pulsational flows as well as monotonic and nonstationary transient regimes have been revealed.

In this work, we present the results of an experimental and theoretical investigation of one of the nonstationary transient regimes, which is a symmetrical (about the vertical axis) four-vortex flow in the form of periodic pulsations of the lower convective cells being partially absorbed by the upper vortices. In a wide range of governing parameters, the structure of this transient flow and its evolution in time have been studied.

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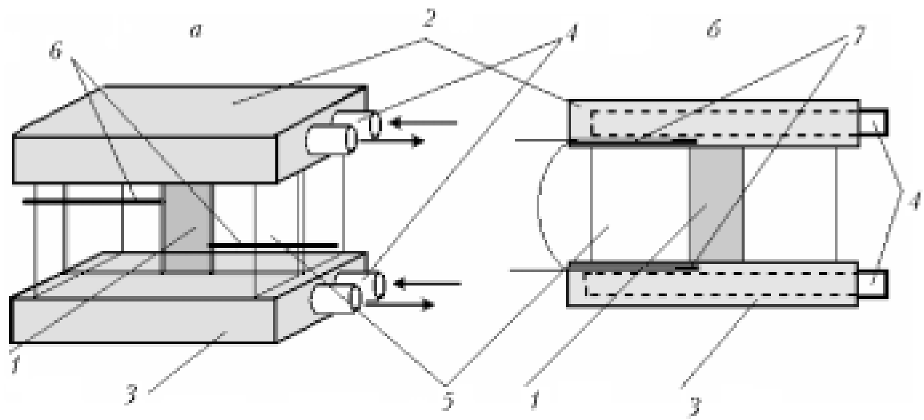

Fig. 1. Schematic diagram of the experimental setup: a) axonometric projection; b) vertical cross section.

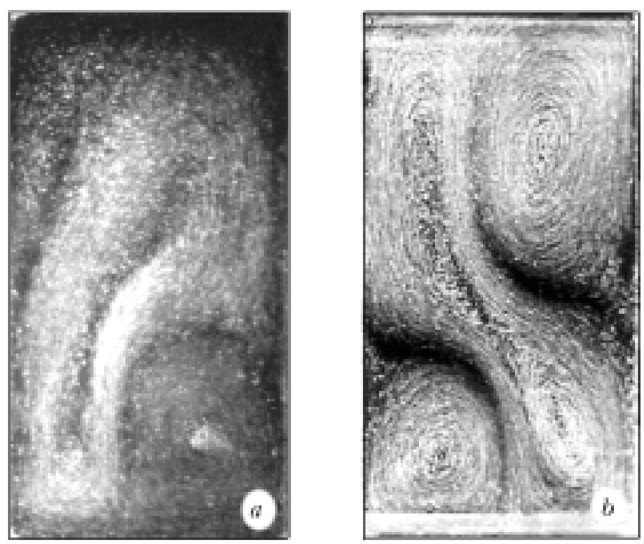

Fig. 2. Stable unsteady-state regimes: a) pulsational regime; b) four-vortex regime with reconnection of corner cells.

Experimental Results. In order to carry out experimental investigations of convective flows in a Hele–Shaw cell, a model was manufactured (Fig. 1). Cell 1 consisted of a cavity of height h = 40 mm, length l = 20 mm, and thickness $2d$ = 2.0 mm, which was bounded from above and below by 16-mm thick aluminum heat exchangers 2 and 3, in which channels of diameter 7.5 mm had been drilled so as to arrange countercurrent flows of a thermostated liquid in the neighboring holes to ensure the temperature homogeneity along the upper and lower boundaries of the cell. Water to the channels was pumped from jet ultrathermostats through branch pipes 4 (the accuracy of sustaining the temperature by the thermostats was $0.05^{o}C$). From the sides, the cavity was bound by a massive piece of plexiglass 5 of length 80 mm and width 50 mm.

The considerable dimensions of the plexiglass block virtually excluded the influence of external thermal conditions; therefore the temperature field was homogeneous horizontally and linear vertically. The working liquid was poured into cell 1 through branch pipes 6 of outer diameter 2 mm, which were clad into elastic rubber pipes. After the cavity had been filled with the liquid, the rubber pipes were pinched at a distance of 50 mm from the branch pipes, and they played the role of pressure dampers.

The difference in temperatures between the heat exchangers was measured by a differential copper-constantan thermocouple, with the constantan wire being of diameter 0.20 mm and the copper one 0.15 mm; the length of the junctions was 1 mm. The thermocouple junctions were inserted into holes 7 drilled in the heat exchangers. The emf

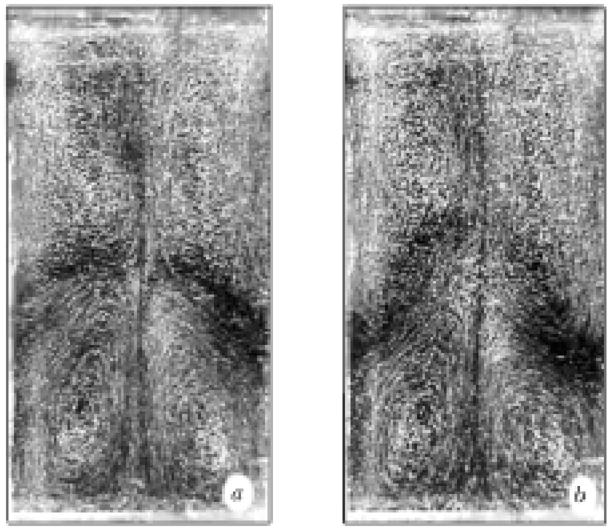

Fig. 3. Characteristic stages of the transient four-vortex regime: a) at the instant of time corresponding to the maximum intensity of upper vortices; b) formation of a convective plume.

was 0.41 μV/K, and it was measured by a V7-54/3 digital voltmeter. The temperature difference was determined within 0.025^{o}C. The temperature drop between the heat exchangers was maintained within 0.05^{o}C.

The experiments have shown that at Ra > 2.1 the pulsational flow (Fig. 2a), which was stable within the range of Rayleigh numbers ΔRa = 1.9–2.1, is replaced by an unstable four-vortex regime with periodical reconnection of the corner vortices (Fig. 2b). The replacement of the pulsational regime begins from upward propagation of the lower cell, as a result of which it begins to occupy a larger and larger region in the cavity. Thereafter, reconnection with the small vortex in the opposite corner occurs, and vibrations set in, the characteristic stage of which is shown in Fig. 2b. Thus, at Ra = 2.2 and 2.4, in the regime of reconnection of vortices the period of vibrations is equal to about 3 min and 1 min, respectively. However, it has been established in the course of the experiments that the attainment of the unstable four-vortex regime may occur in a complicated manner. In particular, under certain conditions a self-oscillating flow of transient type which strongly delayed the emergence into a stable four-vortex regime was observed. Two characteristic stages of such a transient pulsational flow at different instants of time are shown in Fig. 3. The transient regime refers to an unsteady-state flow in the form of four symmetrical (about the reflection in the vertical plane) vortices (Fig. 3a). With time, the lower vortices first increase in dimensions, displacing the upper ones, and then simultaneously begin to decrease with increase in the upper ones. At each instant of time, they differ from each other in both intensity and dimensions. It is seen that the vortices are symmetrical about the reflection in the vertical plane which passes through the middle of the cavity. The twisting of the lower vortices is always such that the liquid ascends along the side walls and descends along the cavity symmetry axis. The liquid ascending along the side walls is declined by the upper oppositely twisted vortices to the symmetry axis and, as a result, a kind of a convective plume is formed in the middle of the cavity (Fig. 3b). At a certain stage this convective plume breaks up, and the picture of convective vortices is repeated. The convective regime presented in Fig. 3 is a transient one: with time it is always rearranged into a stable four-vortex flow with reconnection of the corner vortices. The number of oscillations is changed from four to ten, depending on the value of the Rayleigh number. As the supercriticality increases, such transient flows are observed more often.

Computational Technique. The further investigation performed numerically has confirmed the existence of transient flows of such a type. In the course of calculations, the structure of the four-vortex transient flow was studied depending on the Rayleigh number in a wide range of Prandtl numbers.

In connection with the Hele–Shaw approximation, we assume that the geometrical parameters of the problem satisfy the requirement that h, $l >> d$ (l is the length of the cell). In modeling convection, this restriction on the cell thickness allows one to use an approximation of plane trajectories, according to which convective motions in a liquid are possible only in the plane of wide faces (x, y). This means that the transverse z component of the velocity is equal to zero: $\mathbf{v}(v_x, v_y, 0)$.

In order to theoretically describe convective motions in the Hele–Shaw cell, we will use the classical equations of heat convection in the Boussinesq approximation [5]. In a dimensionless form, this system of equations has the form

$$\frac{\partial \mathbf{v}}{\partial t} + \frac{1}{\mathrm{Pr}} (\mathbf{v}\nabla)\, \mathbf{v} = -\nabla p + \Delta \mathbf{v} + \mathrm{Ra}\, T\gamma\,, \tag{1}$$

$$\mathrm{Pr}\, \frac{\partial T}{\partial t} + \mathbf{v}\nabla T = \Delta T\,, \tag{2}$$

$$\mathrm{div}\, \mathbf{v} = 0\,. \tag{3}$$

In order to express dimensional quantities in dimensionless form, it is adopted that $t = d^2/\nu$ (time), $v = \chi/d$ (velocity), $T = \Theta$ (temperature), and $p = \rho\nu\chi/d^2$ (pressure). As a unit of length we used the half-thickness of the layer d. The equations involve two dimensionless parameters: Rayleigh number Ra and Prandtl number Pr:

$$\mathrm{Ra} = \frac{g\beta A d^4}{\nu\chi}\,, \quad \mathrm{Pr} = \frac{\nu}{\chi}\,.$$

The characteristic temperature gradient A was determined in terms of the difference of temperatures between the heat exchangers Θ and the cavity height: $A = \Theta/h$.

At the solid boundaries of the cavity we set the adhesion conditions for the velocity: $\mathbf{v}|_s = 0$. We will consider a Hele–Shaw cell with thermally insulated vertical faces on which a constant temperature gradient corresponding to heating from below is sustained. The geometry of the problem makes it possible to reduce the three-dimensional problem to a plane one; therefore, a further analysis of thermal convection is made on the basis of the equations written in terms of the stream function and temperature:

$$v_x = \frac{\partial \psi}{\partial y}\,, \quad v_y = -\frac{\partial \psi}{\partial x}\,.$$

Here $\psi(x, y, z)$ is the stream function for the field of the velocity $\mathbf{v}$, the dependence of which on the coordinate z is modeled with the aid of the trigonometric function:

$$\psi\,(x, y, z) = \psi\,(x, y) \cos\,(\pi z/2)\,.$$

From the temperature field we will isolate the equilibrium part $T = T_0 + \theta(x, y)$, where the equilibrium profile of temperature T_0 depends linearly on the vertical coordinate and corresponds to heating from below: $T_0 = -y$. Finally, the system of equations (1)–(3) in terms of ψ and θ has the form

$$\frac{\partial \varphi}{\partial t} + \frac{1}{\mathrm{Pr}} \left(\frac{\partial \psi}{\partial x} \frac{\partial \varphi}{\partial y} - \frac{\partial \psi}{\partial y} \frac{\partial \varphi}{\partial x} \right) = \Delta \varphi - \mathrm{Ra}\, \frac{\partial T}{\partial x}\,, \tag{4}$$

$$\mathrm{Pr}\, \frac{\partial T}{\partial t} + \frac{\partial \psi}{\partial x} \frac{\partial T}{\partial y} - \frac{\partial \psi}{\partial y} \frac{\partial T}{\partial x} = \Delta_1 T + \frac{\partial \psi}{\partial x}\,, \tag{5}$$

$$\varphi = -\Delta_1 \psi\,, \quad \Delta_1 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\,,$$

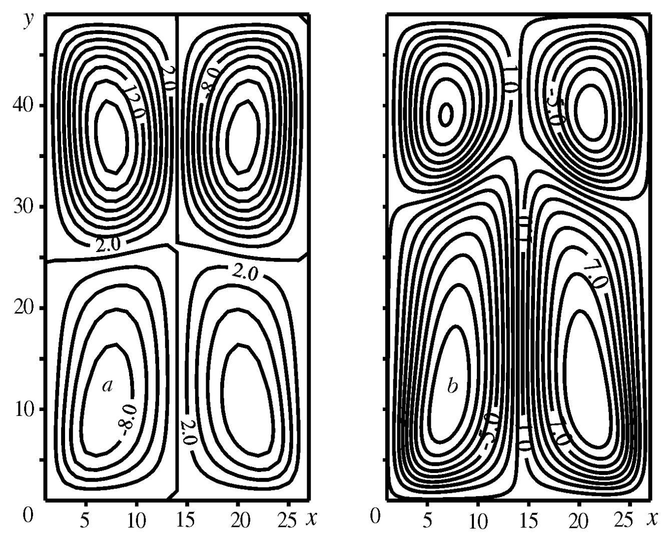

Fig. 4. Stream-function field at different instants of time in the regime in which the upper and lower vortices alternatively displace one another.

where φ is the vorticity and Δ_1 is the plane Laplace operator. The boundary conditions for the system of equations (4)–(5) take the form

$$x = 0\,,\;\; x = L:\;\; \psi = \frac{\partial\psi}{\partial x} = 0\,,\;\; \frac{\partial\theta}{\partial x} = 0\,;\;\; y = 0\,,\;\; y = h:\;\; \psi = \frac{\partial\psi}{\partial y} = 0\,,\;\; \theta = 0\,. \tag{6}$$

The condition of thermal insulation of the wide faces ($\partial\theta/\partial z = 0$ at $z = \pm 1$) is satisfied automatically.

The system of equations (4)–(5) with boundary conditions (6) was solved numerically by the method of finite differences [6]. In the calculations, the two-field method combined with an explicit scheme was used. In approximation of the time and space derivatives, one-sided and central differences were used, respectively. In order to coordinate with the experiment, computations were performed for a cavity with a side ratio of 2 : 20 : 40. The working number of grid nodes in the plane of wide faces was 27 × 49. In calculating the phenomena of capture of convective vortices by the main unsteady-state flow, a finer grid (35 × 65) was used, with the computer module being written in Fortran-90.

Results of Calculations. In numerical modeling, the wide faces are assumed to be thermally insulated, which substantially lowers the threshold of the appearance of all convective regimes. As a result, the pulsational regime (Fig. 2a) is replaced by vibrations (Fig. 2b) at Ra = 1.6.

The calculations show that for a transient flow to be realized in Hele–Shaw cell, the initial conditions symmetrical about reflection in the vertical plane are required. Thus, the possibility of experimentally observing the structures symmetrical about the vertical line for Ra ≥ 2.1 points to the fact that the experimental setup was manufactured according to the specifications without the defects that disturb the symmetry of the cavity. Any asymmetry of the initial conditions prevents observation of this transient flow in calculations — immediately a four-vortex flow with reconnection of vortices appears in the cavity.

In the course of calculations, transient flows of two types were revealed, depending on the values of the Prandtl- and Rayleigh numbers; these flows differ qualitatively from each other. At low Rayleigh numbers, the transient regime is characterized by an alternative mutual displacement of the lower and upper vortices. Two characteristic stages of this regime are presented in Fig. 4 (Pr = 17 (alcohol), Ra = 1.8). The stream-function field depicted in Fig. 4a shows that at a certain instant of time the upper and lower vortices occupy approximately identical regions, but the upper vortices are more intense. It is seen that the lower vortices lift the liquid along the side walls of the cavity, in conformity with experimental data. With time, the intensity of the lower vortices begins to increase. The ascending liquid flow is bent by the upper vortices to the vertical symmetry axis of the cell, which leads to formation of a convective plume in the middle of the cavity (Fig. 4b). At the next stage, the upper cells begin to increase in size; the lower vortices are displaced and they lose "convective tips." The convective system regains its initial state. Such pul-

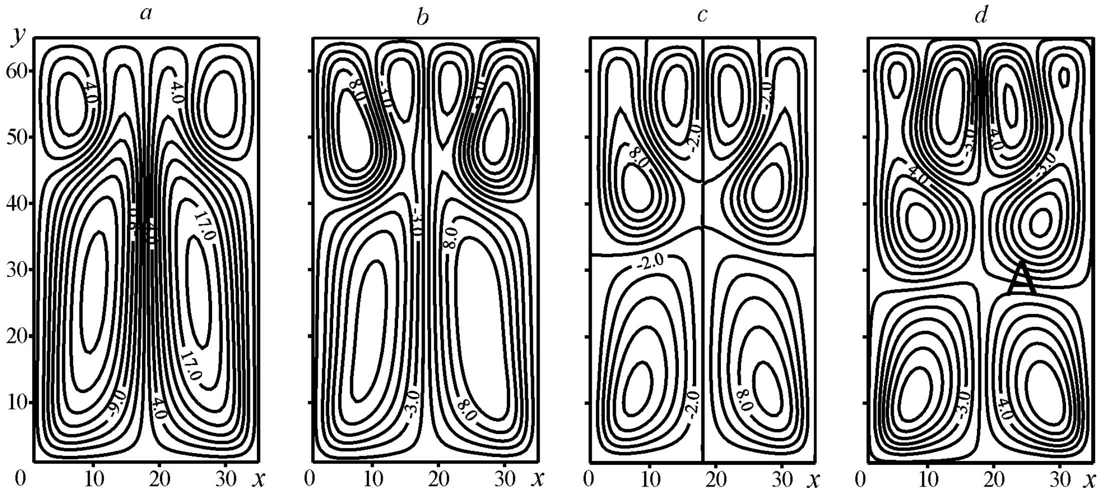

Fig. 5. Stream-function field at different instants of time in the regime of partial break of the lower cells and capture of convective vortices by the upper cells.

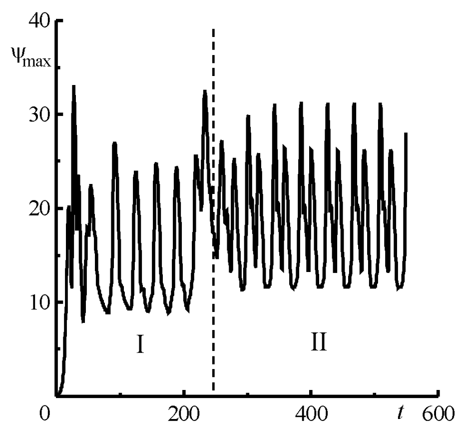

Fig. 6. Time dependence of the maximum value of stream function in the cavity: I) transient regime; II) stable four-vortex regime with reconnection of corner cells.

sations are repeated 5–10 times depending on the Rayleigh number. Thereafter, one of the lower cells joins with the upper one in the opposite corner of the cavity, resulting in the appearance of a stable four-vortex oscillatory regime with reconnection of vortices.

In the region of high Rayleigh numbers, the structure of the transient flow at each instant of time undergoes more considerable changes. Now, the periodic pulsations of the lower convective cells are accompanied by them being partially absorbed by the upper vortices. The characteristic stages of the transient flow for four different instants of time are presented in Fig. 5 (Pr = 6.7 (water), Ra = 2.3). First, the lower vortices dominate over the upper ones, with the convective plume along the vertical symmetry axis of the cavity reaching the upper boundary (Fig. 5a). Thereafter, the upper convective cells begin to grow, and they pinch the convective plume (Fig. 5b). This results in the capture of two small symmetric vortices which are localized near the upper boundary of the cavity (Fig. 5c). The separation of convective vortices leads to a decrease in the kinetic energy of the rotational motion of the lower vortices; therefore their rotation is retarded. The further evolution of the convective system is as follows: the vortices localized near the upper boundary begin to grow, and they displace the oppositely twisted vortices downward (Fig. 5d). The three-story structure is unstable, and, as a result, the oppositely twisted vortices displaced into the middle of the cavity get retarded, forming a stagnation zone. The upper and lower vortices, while rotating in one direction, lift the liquid along the side walls of the cavity. With time, this near-wall ascending flow makes the upper and lower vortices unite. However, in the upper corners of the cavity, small oppositely twisted vortices are formed, which begin to increase in size. Thus, the convective system regains its initial state and the process is repeated.

It is seen from Figs. 2–5 that the transient regimes discussed differ from the steady-state flows not only in shape but also in intensity, and, as calculations show, possess the characteristic features of heat transfer. Thus, in constructing various technical devices and technological installations one must always take into account the possibility of the appearance of transient regimes. The characteristic time dependence of the maximum value of the stream function is presented in Fig. 6 for Pr = 6.7 and Ra = 2.2. It is seen that over time intervals I and II convective flows with different amplitudes are realized. In interval I, a transient flow in the form of two pairs of symmetric vortices is observed. Time interval II corresponds to a stable four-vortex regime with reconnection of corner cells.

Conclusions. An unsteady-state transient regime of thermal convection in a Hele–Shaw cell is investigated experimentally and theoretically. This flow refers to pulsations of two pairs of vortices symmetric about their vertical axis. With time, the symmetry is disturbed, and this leads to rearrangement of the transient flow into a stable four-vortex regime with reconnection of corner convective cells. Under certain conditions, a specific phenomenon of the separation of convective vortices from the main flow is observed, and it was studied theoretically in a wide range of governing parameters. It has been established that the inception of a transient flow leads to a considerable increase in the time needed to attain a stable regime, which should be allowed for in constructing various technical facilities.

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NOTATION

A, characteristic gradient of temperature, K/m; *d*, half-thickness of cavity, m; *g*, free fall acceleration, m/sec^2; *h* and *l*, height and length of the cavity, m; $p(x, y)$, pressure field; Pr, Prandtl number; Ra, Rayleigh number; $T(x, y)$, temperature field; **v**, velocity field; β, coefficient of thermal expansion, 1/K; γ, unit vector directed vertically upward; Θ, characteristic difference of temperatures on heat exchangers, K; θ, deviation of temperature from the equilibrium profile; ν, coefficient of kinematic viscosity, cm^2/sec; ρ, density of liquid, kg/m^3; $\varphi(x, y)$, vorticity field; χ, coefficient of thermal diffusivity, cm^2/sec; $\psi(x, y)$, stream function for averaged velocity. Subscript: max, maximum.

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